Warm-up!

$f(4) = \frac{0}{13} = 0$

$\frac{(x+2)}{(x+2)^2} = \frac{1}{x+2}$

Let $f(x) =$ $(x$

A) What feature does the graph have at

hole $(-2, \quad)$

B) What feature does the graph have at

Vertical Asymptote $x = -\frac{1}{3}$

C) What feature does the graph have at

x-intercept $(4, 0)$

Unit 1 Day 9 (1.8+1.11)

- Rational Function Zeros
- Long Division
- Crossing Horizontal/Slant Asymptotes

Homework: Zeros and Long Division

Quiz Tues/Weds (Sept 9th/10th)

What's going on at $x = c$

Let $f(x) = \frac{n(x)}{d(x)}$

If $n(c) \neq 0$ and $d(c) = 0$,
$x = c$ is a vertical asymptote

$$y = \frac{(x+3)(x-5)}{(x+3)(x+7)}$$

$x = -7$ is a vertical asymptote

If $n(c) = 0$ and $d(c) = 0$,
Simplify to find out

$$y = \frac{(x+3)(x-5)}{(x+3)(x+7)}$$

There is a hole at $(-3, -2)$

$$y = \frac{(x+3)(x-5)}{(x+3)^2(x+7)}$$

$x = -3$ is a vertical asymptote.

If $n(c) = 0$ and $d(c) \neq 0$,
$x = c$ is a zero of $f(x)$.
Graphically, this means $(c, 0)$ is an x-intercept.

$$y = \frac{(x+3)(x-5)}{(x+3)(x+7)}$$

$x = 5$ is a zero
$(5,0)$ is an x-intercept

Find all holes, x-intercepts, and vertical asymptotes for the following functions

$$f(x) = \frac{x^3 - 2x^2 + x}{9x^4 - 16x^2}$$

$$f(x) = \frac{(x)(x^2-2x+1)}{(x)^2(9x^2-16)}$$

$$f(x) = \frac{\cancel{(x)}(x-1)^2}{(x)^{\cancel{2}}(3x-4)(3x+4)} = \frac{(x-1)^2}{(x)(3x-4)(3x+4)}$$

X-intercept: $(1,0)$

VA: $x=0$ $\quad x=\frac{4}{3}$ $\quad x=-\frac{4}{3}$

Hole: None

$$g(x) = \frac{x^3 + 6x^2 + 9x}{5x^2 + 13x - 6}$$

$$g(x) = \frac{x(x^2+6x+9)}{(5x-2)(x+3)}$$

$$g(x) = \frac{(x)(x+3)^{\cancel{2}}}{(5x-2)\cancel{(x+3)}} = \frac{(x)(x+3)}{5x-2}$$

X-intercept: $(0,0)$

VA: $x=\frac{2}{5}$

Hole: $(-3,0)$

Hole vs. x-Intercept vs. Vertical Asymptote

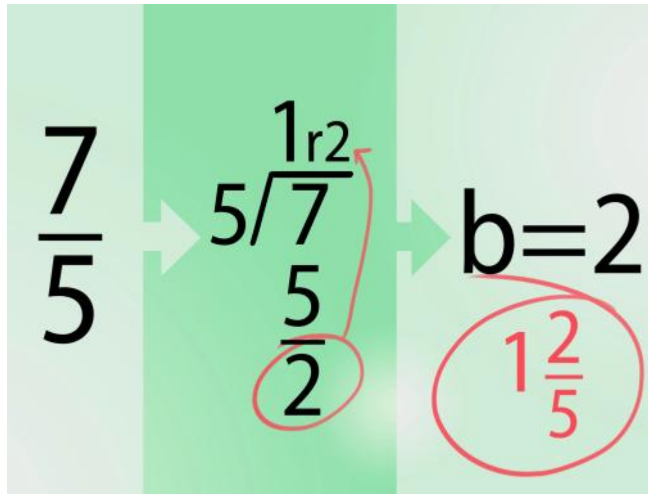
Write a function for which $x = 4$ is a vertical asymptote, $x = 2$ and $x = -1$ are zeros, and there is a hole at $x = 3$

$$\frac{(x-2)(x+1)(x-3)}{(x-3)(x-4)}$$

Polynomial (Long) Division

Divide the quotient to rewrite $f(x)$ in an equivalent form.

$$f(x) = \frac{x^2 - 8x + 23}{x - 5} = x - 3 + \frac{8}{x-5}$$



$$\begin{array}{r|l}
 & x - 3 \\
\hline
x - 5 & x^2 - 8x + 23 \\
 & -(x^2 - 5x) \\
\cline{2-2}
 & -3x + 23 \\
 & -(-3x + 15) \\
\cline{2-2}
 & 8
\end{array}$$

Slant Asymptote:

$y = x - 3$

$$\begin{array}{r|rrr}
5 & 1 \; x^2 & -8 & 23 \\
 & & 5 & -15 \\
\hline
 & 1 \; x & -3 & + \frac{8}{x-5}
\end{array}$$

Divide the quotient to rewrite $g(x)$ in an equivalent form.

$$g(x) = \frac{10x - 19}{2x - 3} = 5 + \frac{-4}{2x-3}$$

$$5 - \frac{4}{2x-3}$$

$$\begin{array}{r|l} & 5 \\ 2x-3 & 10x - 19 \\ & \underline{-(10x - 15)} \\ & -4 \end{array}$$

Polynomial (Long) Division

Divide the quotient to rewrite $h(x)$ in an equivalent form.

$$h(x) = \frac{x^2 + 2x - 7}{x + 3} = x - 1 + \frac{-4}{x+3}$$

$$x - 1 - \frac{4}{x+3}$$

$$\begin{array}{r}
x \quad -1 \\
x+3 \enclose{longdiv}{x^2 + 2x - 7} \\
-(x^2 + 3x) \\
\hline
-x - 7 \\
-(-x - 3) \\
\hline
-4
\end{array}$$

Divide the quotient to rewrite $k(x)$ in an equivalent form.

$$k(x) = \frac{3x^2 - 17x - 23}{x^2 - 6x - 7}$$

Polynomial (Long) Division

$y = -2x + 1$

Determine the equation of the slant asymptote.

$$y = \frac{-2x^3 + 5x^2 - 5x - 3}{x^2 - 2x + 1} = -2x + 1 + \frac{-x - 4}{x^2 - 2x + 1}$$

$$-2x + 1 - \frac{x + 4}{x^2 - 2x + 1}$$

$$
\begin{array}{r}
-2x + 1 \\
x^2 - 2x + 1 \enclose{longdiv}{-2x^3 + 5x^2 - 5x - 3} \\
-(-2x^3 + 4x^2 - 2x) \\
\hline
x^2 - 3x - 3 \\
-(x^2 - 2x + 1) \\
\hline
-x - 4
\end{array}
$$

$$f(x) = \frac{x^2 - 3x + 2}{x + 3}$$

Vertical asymptote(s):

Horizontal asymptote:

Slant asymptote:

Hole(s):

x-intercept(s):

y-intercept:

$\lim_{x \to \infty} f(x) =$

$\lim_{x \to -\infty} f(x) =$

$h(0) = \frac{24}{-9} = -\frac{8}{3}$

$$h(x) = \frac{-(x+4)(x-6)}{x^2-9} = \frac{-x^2+2x+24}{x^2-9}$$

$$-\frac{(x+4)(x-6)}{(x+3)(x-3)}$$

Vertical asymptote(s): $x=3$ and $x=-3$

Horizontal asymptote: $y=-1$

Slant asymptote: None

Hole(s): None

x-intercept(s): $(-4,0)$ $(6,0)$

y-intercept: $\left(0, -\frac{8}{3}\right)$

$\lim_{x\to\infty} f(x) = -1$

$\lim_{x\to-\infty} f(x) = -1$

$$\begin{array}{r|l} & -1 \\ x^2+0x-9 & -x^2+2x+24 \\ & -(-x^2+0x+9) \\ \hline \end{array}$$

$$k(x) = \frac{x+5}{x^2+4x+8}$$

Vertical asymptote(s):

Horizontal asymptote:

Slant asymptote:

Hole(s):

x-intercept(s):

y-intercept:

$\lim_{x\to\infty} f(x) =$

$\lim_{x\to-\infty} f(x) =$

$$g(x) = \frac{3x^2 - 5x - 2}{x - 2}$$

Vertical asymptote(s):

Horizontal asymptote:

Slant asymptote:

Hole(s):

x-intercept(s):

y-intercept:

$\lim_{x \to \infty} f(x) =$

$\lim_{x \to -\infty} f(x) =$

Determine a rational function $f(x)$ with the following properties

- The only zeros are $x = 1$ and $x = 3$
- There is a hole at $x = -2$
- $\lim_{x\to\infty} f(x) = -\infty$
- $\lim_{x\to-\infty} f(x) = -\infty$